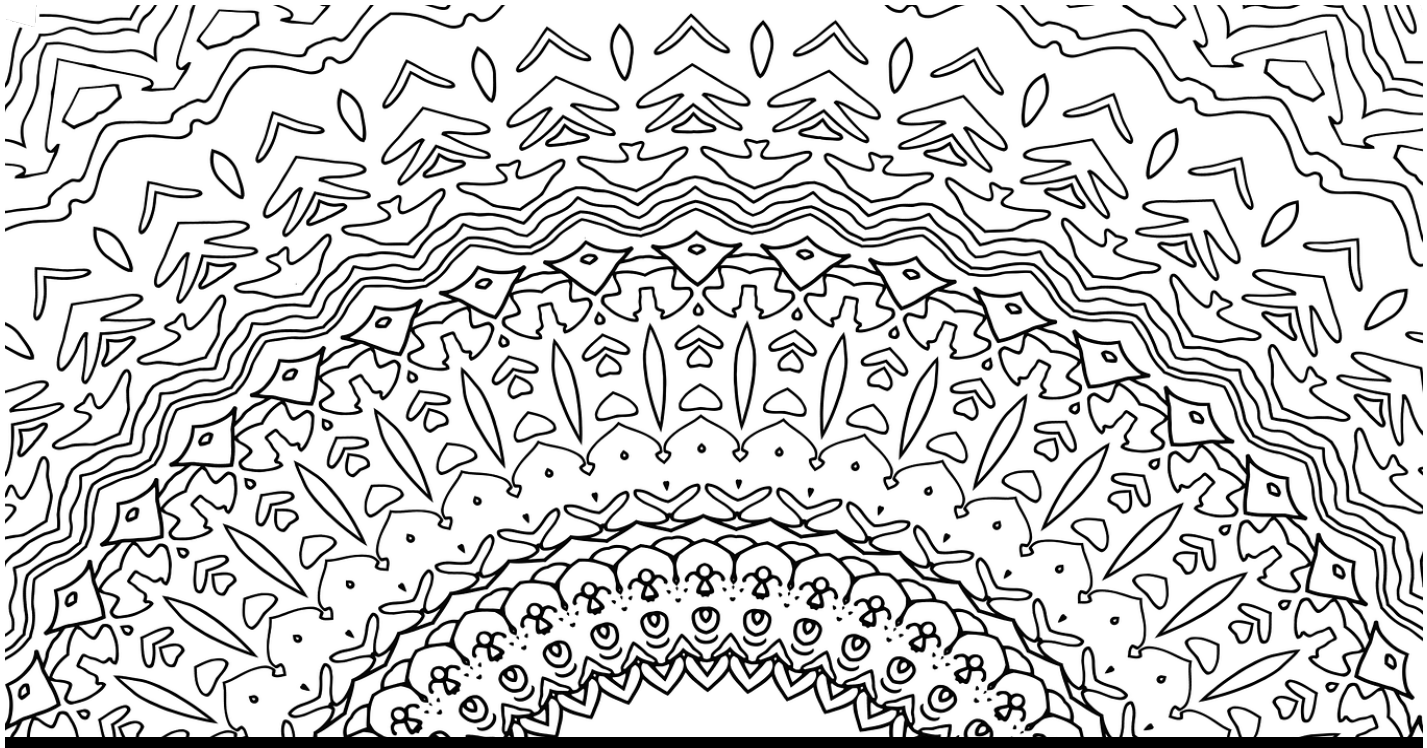

SUMMER REVIEW PACKET:

MATH SKILLS

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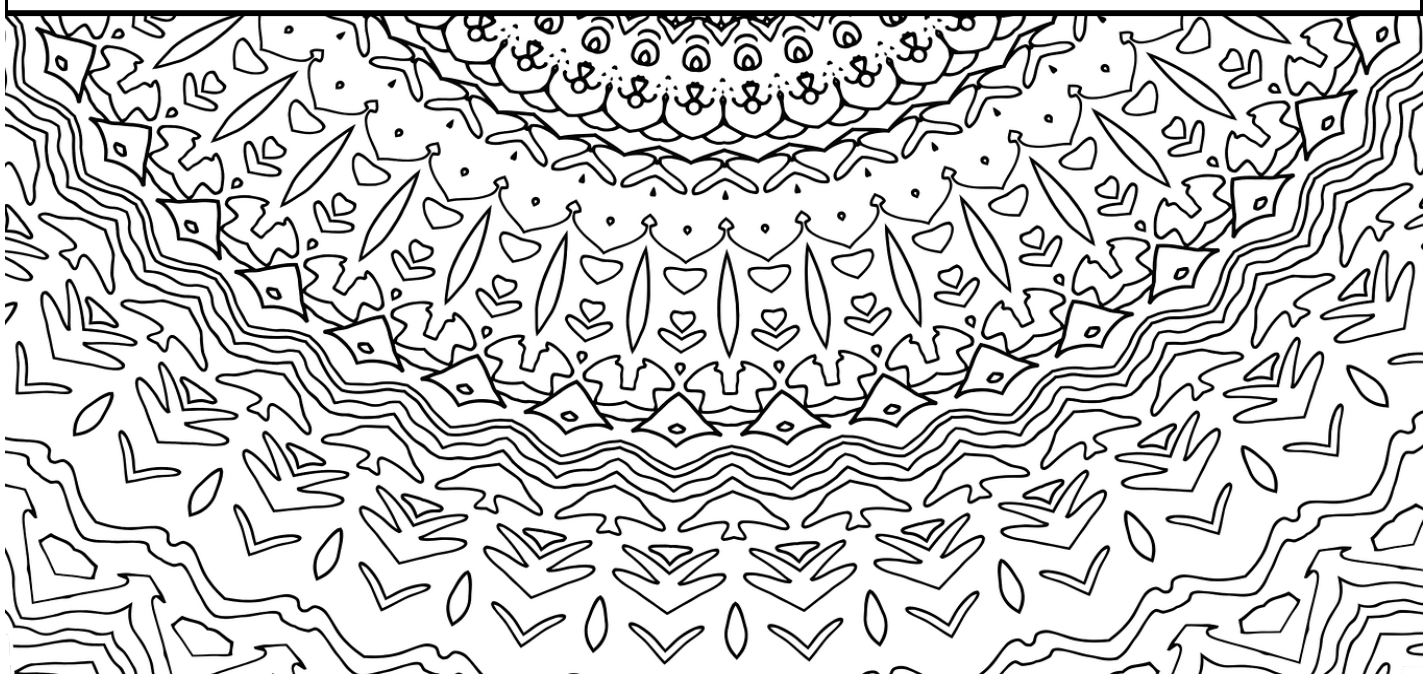


Math Summer Review

Use this packet to brush up on the skills you need for upper level math and science courses. This review is broken down into two portions; Cornell notes based on topic, and lessons with practice problems. Be sure to do all work before checking your answers. When you do check your answers, if you got the problem wrong, do a little error analysis to find where you went wrong and figure out how to correct it. You may want to do correction in a different color or on a separate paper.

Begin each lesson by completing the appropriate Cornell notes. Do NOT rush through all the Cornell notes and then try to complete the lessons separately. Complete the Cornell notes and associated lessons at the same time to help you truly understand, grasp, and practice the concepts. You will also want to add more than just the information found in the links below. There are additional links and information in each lesson. Be thorough in your work

Cornell Notes Tutorial Links:

Algebra Basics

https://www.khanacademy.org/math/algebra-basics

Algebra

https://www.khanacademy.org/math/algebra

Algebra 2

https://www.khanacademy.org/math/algebra2

Trigonometry

https://www.khanacademy.org/math/trigonometry

Significant Figures

https://www.youtube.com/watch?v=hQpQ0hxVNTg

Metric Units & Conversions

https://www.youtube.com/watch?v=uHaKyNplino&t=3s

Units of Measurement

http://www.npl.co.uk/upload/pdf/units-of-measurement-poster.pdf

What is Algebra?

Algebra is a branch of mathematics that substitutes variables (letters) for unknown numbers. These variables are solved for by applying mathematical knowledge and concepts. This workshop will guide you through the main algebraic concepts needed to succeed in your next math course or even upper level science.

The concepts you learn in algebra will be used across many other subjects. For example, you will use algebra in chemistry, physics, trigonometry, and many other upper level science and math courses.

Lesson 1: The Basics of Algebra

Order of operations is your Golden Rule! Remember back in elementary school, "Please excuse my dear aunt sally"? It's as simple as that. Begin with parentheses, next exponents or square roots, then multiplication, then division, and then addition, lastly subtraction.

Example: PEMDAS

$7 + (6 \times 5^2 + 3)$=?

PEMDAS tells us to begin with the numbers in the parentheses and the exponent.

$7 + (6 \times 25 + 3)$=?

Next, PEMDAS tells us to do multiplication.

$7 + (150 + 3)$=?

Now, we just finish the addition.

$7 + (6 \times 5^2 + 3)$=**160**

Example: A ball is thrown straight up in the air with a velocity of 15m/s, how far did it go in 2.5s?

$height = vt - \frac{1}{2}9.8t^2$

Plug in the values.

$height = (15)(2.5) - \frac{1}{2}9.8(2.5)^2$

PEMDAS tells us to start with the exponent.

$height = (15)(2.5) - \frac{1}{2}9.8(6.25)$

Next, we move on to the multiplication.

$height = 37.5 - 30.625$

Lastly, we complete the subtraction.

$\boldsymbol{height = 6.875\text{m}}$

Most equations you will be solving involve solving for one variable or one unknown. In a later lesson, we will get to tackling solving equations with more than one variable…fun. For now, we will focus on just isolating one variable at a time. For this portion, we will continue to use x as the variable, but be aware that many equations, such as physics equations, use many variables including some from the Greek alphabet.

We will be using several physics equations to practice these algebraic concepts, such as, the example problem above. These equations are perfect for PEMDAS and provide a real-life scenario to solve for.
Now, let's practice isolating a variable. There several algebraic rules to follow in order to complete this task. The following examples will bring you through the most common of these rules.

Example: Isolating x

$3x + 8 = 14$
Subtract the 8 from 14

$3x = 6$
Divide the by 3 on both sides to isolate x
*Remember, in order to isolate a variable, you have to do the opposite mathematical operation for each step!

$\boldsymbol{x = 2}$

Example: Cross Multiplying

$\frac{2}{x} = \frac{12}{1}$
Cross multiply, bring the denominator of one side into the numerator of the other.

$2 = 12x$
Now, divide by 12 on both sides to isolate x.

$\boldsymbol{x = 0.167}$

Example: Square Roots

$6 = \sqrt{x + 4}$
Begin by squaring both sides to get rid of the square root.

$6^2 = (\sqrt{x + 4})^2$
$36 = x + 4$
Now, we can subtract 4 on each side.

$\boldsymbol{x = 32}$

Example: Distributive Property

$3(x + 2) = 12$
Distribute (multiply) the 3 to everything inside the parentheses.

$3x + 6 = 12$
Combine like terms.

$3x = 6$
Now, divide by 3 on both sides.

$\boldsymbol{x = 2}$

A few tips to remember:

- Combine like terms.
- Do the opposite mathematical operation on each side to isolate the variable. One step at a time!
- When two fractions are equal to one another, cross multiply.
- To get rid of a square root, you need to square both sides!
- Using distributive property
    - The distributive property is an algebraic property which is used to multiply a single term into two or more terms inside a set of parentheses.
    - Watch the following video to help remember this property:

      https://www.khanacademy.org/math/pre-algebra/pre-algebra-arith-prop/pre-algebra-ditributive-property/v/the-distributive-property

Practice Problems: Solve for x.

1. $42x - 16 = 83$
2. $7(x + 2) = 32$
3. $x = (2 \times 5) + \frac{1}{2}(9.8)4^2$
4. $\frac{4}{x} = \frac{1}{12}$
5. $2.5x + 55 = 8.75x$
6. $6(x + 5) = 24$
7. $\frac{1}{x} = \frac{2}{3} + \frac{3}{4}$
8. $32 = \frac{x-8}{2}$
9. $5.5x - 3.5 = 25x$
10. $25 = \sqrt{\frac{x}{4}}$

Lesson 2: Converting Words to Math

Algebra has a language all its own. In order to do well in your math and upper level science, you must be able to translate the words into mathematical operations. Let's begin with an introduction to this language.

The following are a list of common terms used to describe the specified operations

Addition

- sum
- more
- more than
- increased by
- exceeds
- total
- net

Subtraction

- difference
- fewer than
- less than
- decreased by
- spends/ loses
- change in
  - Final – Initial

Multiplication

- product
- times
- twice
- each
- of
- by a factor

Division

- quotient
- half
- one-third
- one-fourth
- per
- percent
- out of

Practice: Complete the following table by writing out a mathematical equation for the words. Set these equations equal to x as though you are solving for an unknown.

There is a change in speed from 5mph to 2mph as a car slows.	
The ball bounced 4 times higher when a larger force was applied.	
A frog jumps half way as far as his record of 30ft.	
A student received 77 out of 100 possible points.	
The net force of a mover pushing a sofa is 25N of applied force and -10N of frictional force.	
An apple falls from a tree that is 22m high. The apple lands on-fourth of the way down in a bird's nest.	
The sum of deposits to my bank account are $25, $46, $12, and $31.	
A race has a difference in initial time of 2s and final time of 13s.	
The speed is 45 miles per 1 hour.	
The velocity is increased by a factor of 15.	

Lesson 3: Solving for Other Variables

Now we will use our knowledge of isolating variables and apply it to equations with non-x variables. These are all physical science and physics equations. Don't let the appearance of the equation make you nervous. Yes, they look complicated, but take a good look. Each variable represents a number. So in actuality, these are the same types of equations you've solved in algebra all along. Refer to the following examples and rules in Lesson 1 to help you through this section.

Example: Rearrange the equation to solve for v_i.

$a = \frac{v_f - v_i}{t}$

Here we go! Begin by multiplying both sides by t in order to get rid of the denominator.

$at = v_f - v_i$

Now it's smooth sailing. Subtract by v_f on both sides.

$at - v_f = -v_i$

In order to remove the (-) from v_i, you will need to divide by a -1 on both sides.

$\boldsymbol{v_i = -at + v_f}$

***One additional tip to remember...if you divide by a fraction, you must flip the fraction and multiply by its reciprocal.**

Practice Problems: Rearrange each equation to solve for the specified variable.

1. Solve for m
   $D = \frac{m}{V}$

2. Solve for t
   $v = \frac{x}{t}$

3. Solve for x_f
   $v = \frac{x_f - x_i}{t}$

4. Solve for v (notice the v is not squared!)
   $a_c = \frac{v^2}{r}$

5. Solve for v (notice the v is not squared!)
   $KE = \frac{1}{2}mv^2$

6. Solve for r (notice the r is not squared!)
   $F_g = G\frac{m}{r^2}$

7. Solve for q_1

$$F_{electric} = k\frac{q_1 q_2}{r^2}$$

8. Solve for k

$$PE_{elastic} = \frac{1}{2}kx^2$$

9. Solve for t

$$\Delta x = v_i t$$

10. Solve for a

$$v_f^2 = v_i^2 + 2a\Delta x$$

Lesson 4: Graphing

Types of Graphs:

Graphs are the method used to present data. This helps visualize what the data represents. It is very important that you choose the right type of graph for the type of data you have collected. Here is a breakdown of the 3 most common forms of graphs.

Line Graphs:

This type of graph is a create visual to show changes over time. For most science courses, this is the graph of chose!

Pie Graphs/ Charts:

This type of graph shows parts of a whole broken into percentages to represent pieces of the pie.

Bar Graphs:

This type of graph is used to compare between different groups or track changes. Typically has multiple sets of data displayed. Useful for weather patterns, surveys, etc.

***Not sure which graph really works for you??? Check out this website for help:**

http://www.sciencebuddies.org/science-fair-projects/project_data_analysis.shtml#keyinfo

Of course, as with all things science, there are rules to follow. Here is the general set of rules to learn and memorize for constructing proper graphs.

Rules for Graphing:

1. Draw the axes. Use a straight edge for all parts of the graph.
2. Plot and draw the data/ graph in pencil; this way mistakes can be corrected.
3. The x-axis is the INDEPENDENT variable; time or temperature
4. The y-axis is the DEPENDENT variable; whatever is changing or depends on the independent variable.
5. Label each axis with the data it represents and the unit; include a logical number scale that can clearly show the trend in data.
   - For example, if the numbers in your data are close to one another, use decimals in the scale to spread the data plots out.
6. Always title the graph; it can be just the x- and y-axis vs. each other
   - Example: velocity vs. time; dependent vs. independent
7. Include a legend for graphs that represent multiple pieces of data or trials.

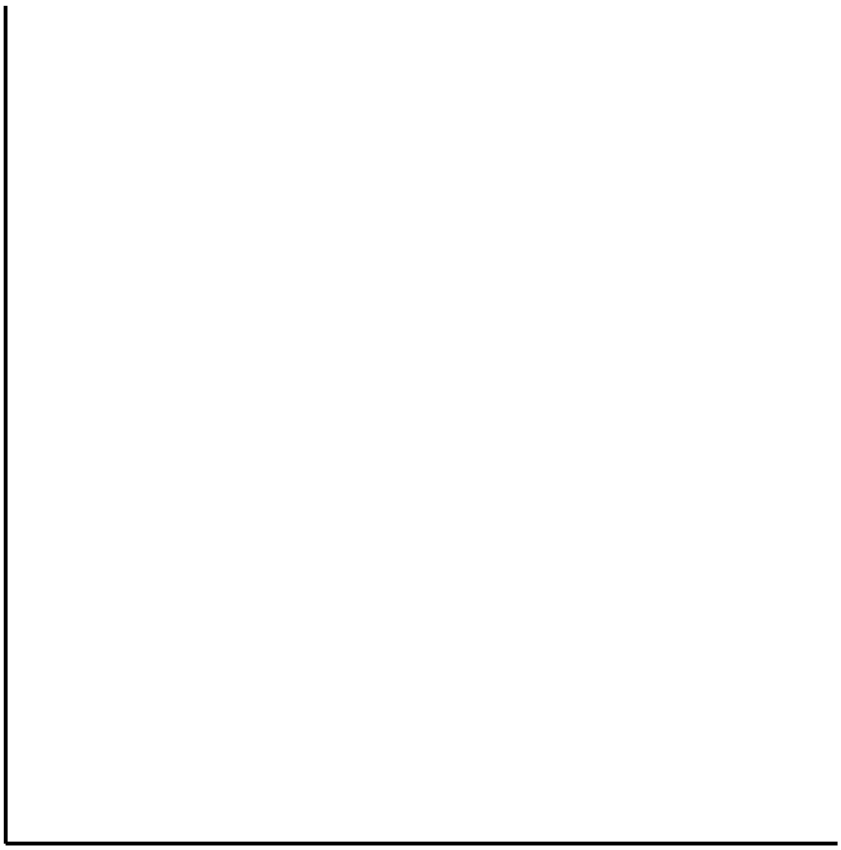
***Still unsure? Or do you want to see these rules in action? Check out this website:**

https://magoosh.com/hs/act/act-study-tips/2016/act-science-graphs-and-tables/

Your turn!

Use the following data table to construct a line graph that follows the graphing rules.

Time (seconds, s)	Position (meters, m)
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16
4.5	20.25
5	25



Lesson 5: Trigonometry

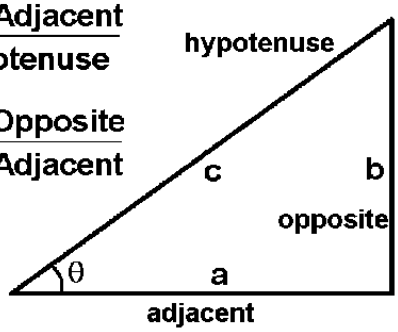
Trigonometry is defined as the branch of mathematics that deals with relations of sides and angles of right triangles. Trigonometry is extremely useful in a physics course. By using trigonometric functions in combination with the Pythagorean theorem, you are able to calculate or even predict values of force, displacement, velocity, and acceleration.

The basic and most commonly used functions are as follows:

$$\text{SIN}\,\theta = \frac{\text{Side Opposite}}{\text{Hypotenuse}}$$

$$\text{COS}\,\theta = \frac{\text{Side Adjacent}}{\text{Hypotenuse}}$$

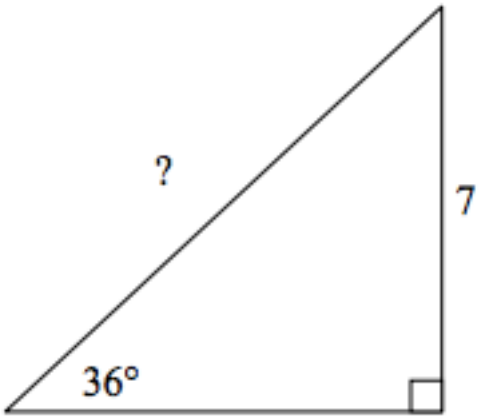
$$\text{TAN}\,\theta = \frac{\text{Side Opposite}}{\text{Side Adjacent}}$$



The Pythagorean theorem is:

$$a^2 + b^2 = c^2$$

Examples: Memorize SOH-CAH-TOA

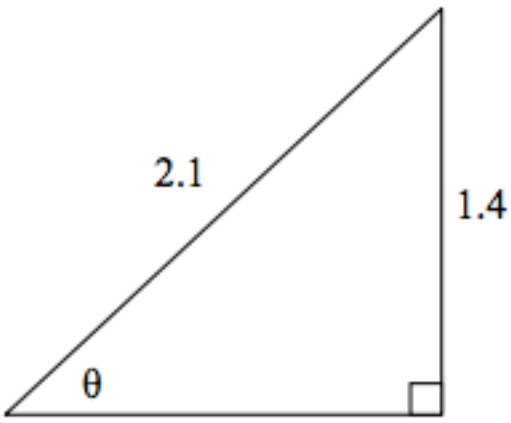


In order to solve this problem, you need to analyze what you are given and what you are trying to find. Since we are given the opposite side of the specified angle and asked to find the hypotenuse, we should use sin.

$$\sin 36 = \frac{7}{x}$$

Now, rearrange and solve for x.

$$x = \frac{7}{\sin 36} = 11.91$$



In order to solve this problem, you need to analyze what you are given and what you are trying to find. We are given the opposite side of theta and the hypotenuse. We are asked to find the value of theta. To do this, we should use sin.

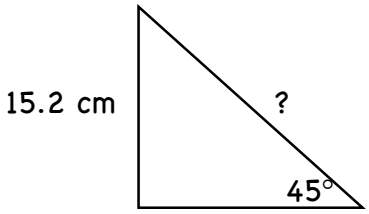
$$\sin\theta = \frac{1.4}{2.1}$$

Now, rearrange and solve for θ by using the inverse of sin.

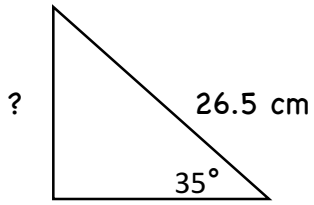
$$\theta = \sin^{-1}\frac{1.4}{2.1} = 41.81°$$

Practice Problems: Complete the following and keep calculator in degree mode.

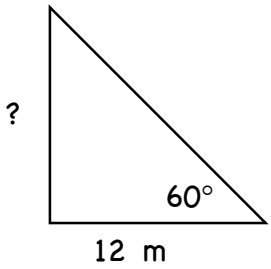
1. Solve for the missing hypotenuse.



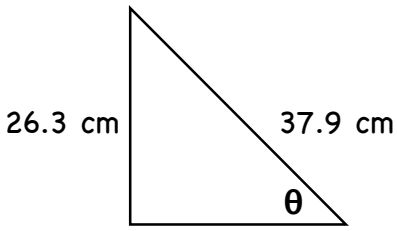
2. Solve for the missing side.



3. Solve for the missing side.

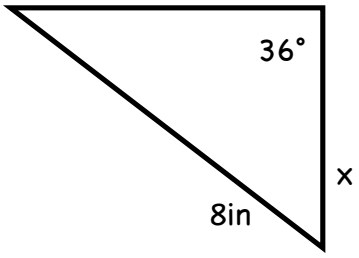


4. Solve for the missing angle.



In the next set of problems, solve for BOTH missing variables using a combination of trig functions only.

5.



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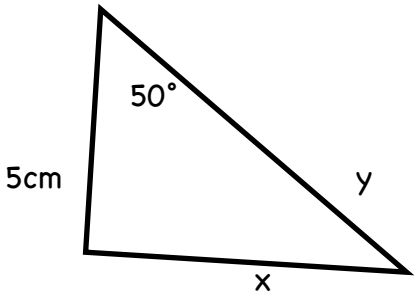

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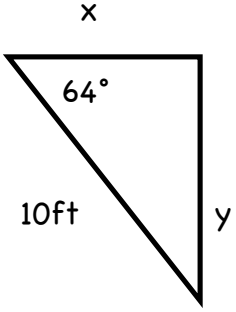

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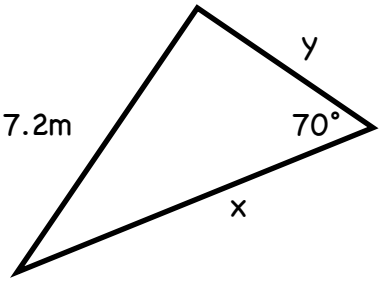

Lesson 6: Significant Figures

Significant figures are an important skill to learn and master. This enables you to record and display more accurate numbers in your calculations. Follow the rules below and answer the following questions.

1) All digits 1-9 inclusive are significant.
   Example: 129 has 3 significant figures.
2) Zeros between significant digits are always significant.
   Example: 5,007 has 4 significant figures.
3) Trailing zeros in a number are significant only if the number contains a decimal point.
   Example: 100.0 has 4 significant figures.
   100 has 1 significant figure.
4) Zeros in the beginning of a number whose only function is to place the decimal point are not significant.
   Example: 0.0025 has 2 significant figures.
5) Zeros following a decimal significant figure are significant.
   Example: 0.000470 has 3 significant figures.
   0.47000 has 5 significant figures.

Significant Figures and Scientific Notation

1. How many significant figures are there in each of the following:

   a. 436.12 _____
   b. 1,000 _____
   c. 203 _____
   d. 223,000 _____
   e. 4,000,000 _____
   f. 53.8 _____
   g. 0.0000084 _____
   h. 0.005501 _____
   i. 5,320. _____
   j. 902.001 _____
   k. 7.5×10^3 _____
   l. 0.0010100 _____

2. Convert the following either from common notation to scientific notation or vice versa.

   a. 43,000,000 _____
   b. 0.000193 _____
   c. 0.0050715 _____
   d. 8.45×10^5 _____
   e. 5.057×10^{-7} _____
   f. 1.486×10^2 _____

3. Solve the following and round your answers to the correct number of significant figures.

   a. (28.3m)(72m) _____
   b. 14.5m + 4.002 m + 12.2 m _____
   c. (361cm)(53.11cm) _____
   d. 4,200kg ÷ 52L _____
   e. 86.2m ÷ 52.781m _____
   f. $(3.01 \times 10^{19}\text{m})(21\text{m})(2.33 \times 10^{-24}\text{m})$ _____
   g. $97.05\text{cm}^2 - 42.034\text{cm}^2$ _____
   h. $(854.1\text{m})(4.050 \times 10^3\text{m})(631.9\text{m})$ _____

Lesson 7: Metric and SI Units & Conversions

The metric unit is based on powers of tens. The special system of measurements used in science is called the SI base unit system. It is based on the metric system. The purpose of using a standard system of measurements is to maintain accuracy, reproducibility, and understanding of data. Remember, not all scientists speak the same language, but the math is universal.

Solve the following problems, expressing the answers in the proper number of sig. figs.

1. What is the sum of 31.5mm, 65.2cm, and 3.28m? Write your final answer in meters.

Metric Prefixes		
Tera	10^{12}	T
Giga	10^{9}	G
Mega	10^{6}	M
Kilo	10^{3}	k
Hecto	10^{2}	h
Deca	10^{1}	da
Deci	10^{-1}	d
Centi	10^{-2}	c
Milli	10^{-3}	m
Micro	10^{-6}	μ
Nano	10^{-9}	n
Pico	10^{-12}	p

2. If 43.5L of propane is drawn from a tank originally containing 44.3L of propane, what volume of propane remains in the tank?

3. What is the area of the bottom of a box measuring 35.0cm long and 15.0cm wide?

4. How many centimeters are there 62 inches? (Hint 1in=2.54cm)

5. What is the distance, in kilometers, of a 4.5mile cross-country course? (Hint: 1mile=1.60934km)

6. What is the density of a substance that measures 55.03g with volume of 24L? (Hint: density is the quotient of mass over volume)

7. Convert each of the following measurements into meters:

   a. 3.2 nm

   b. 65 km

   c. 7.3 pm

   d. 315 μm

8. Rank the following mass measurements from smallest to largest: (Hint: convert to grams first!)
   23.6 mg 1,034 μg 0.000055 kg 0.24 mg

9. Convert each of the English quantities to their metric equivalents: (Hint:1in=2.54cm)
   a. 584 ft. to m

   b. 9.0 in. to mm